Modelling beach-structure interaction using a Heaviside technique: Application and validation

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ABSTRACT

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In this study, an analytical solution, based on a Heaviside technique, is developed to model the shoreline evolution in the vicinity of a groyne due to a random sequence of waves. The beach at Borth, Wales, UK was used as a case-study. A wave time-series covering a time period of about 12 years, was used to test the performance of a recently constructed coastal defence scheme. Transformations of the wave time-series from offshore to nearshore were performed using a semi-empirical procedure. Three different wave breaking formulae were independently applied to the wave model, and their effects to the consequent shoreline evolution were investigated. In addition, three different longshore transport formulae were compared. These were the CERC, the Kamphuis and the Bayram formulae. Results showed that the CERC formula predicted a significantly greater amount of sediment transport and hence erosion on the downdrift side of the groyne while the models based on Kamphuis and the Bayram formulae gave comparable results. All the results exhibited a strong sensitivity to the temporal resolution of the forcing. Finally, some sensitivity to the treatment of wave breaking was found.

ADDITIONAL INDEX WORDS: *shoreline evolution, analytical model, groyne, waves, jetty, longshore sediment transport rate, wave breaking conditions, erosion, accretion .*

INTRODUCTION

Beach erosion is a very significant topic in the field of coastal engineering. The main reason is that this physical phenomenon, under certain circumstances, can become a serious threat for properties, structures and activities which take place in the coastal zone. One such a case is the coastal area of Borth in Wales, UK.

Being renowned for its beautiful sandy beach, which attracts visitors, surfers and bathers, the coast of Borth is threatened by waves propagating both from the southwest and the northwest (Figure 1). In order to protect the beach from wave-induced erosion, a coastal defence system comprising a set of groynes and breastworks directly fronting the village had been constructed 40 years ago. However, this defence system exceeded its useful life and as a result, a new one was constructed. The new coastal defence scheme involves two rock breakwaters, two rock groynes, and a multi-purpose reef.

We were motivated by the need to investigate the performance of the new defence scheme with the help of analytical modelling. For this purpose, the new coastal defence system was represented by a single groyne so as to assess the impacts of the blockage of littoral drift. Previous analytical solutions regarding the erosion/accretion phenomenon on a beach near a groyne have been presented e.g. by Larson *et al.* (1987), and Larson *et al.* (1997). However, they were limited to the specific case of constant wave characteristics of breaking waves. Analytical treatments of time-varying conditions were introduced by Larson *et al* (1997) who considered a sinusoidal variation in wave direction. Reeve (2006) and Zacharioudaki and Reeve (2008) developed integral solutions for waves with arbitrarily varying height, period and direction while Walton and Dean (2011) presented closed-form solutions for piece-wise constant wave conditions. In this study we investigate the application of the piece-wise approach to a coastal scheme at Borth, Wales, UK.

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METHODOLOGY

The linearized partial differential equation which expresses the shoreline evolution caused by the impact of wave action, (which has the form of a diffusion equation), is produced by coupling the longshore sediment transport rate equation, and the continuity of sand equation (Equation 1), which may be written as:

$$\frac{\partial y}{\partial t} + \frac{1}{(h_* + B)}\frac{\partial Q}{\partial x} = 0 \tag{1}$$

where y is the cross-shore position of the shoreline, x is the longshore distance, h_* is the depth of closure, B is the berm height and Q is the longshore transport rate, see eg.; (Kamphuis, 2000, Dean and Dalrymple, 2002, Reeve *et al.*, 2004). A well-known formula is the CERC equation, with Q given by the following expression:

$$Q = \frac{\rho K \sqrt{g/\gamma_b}}{16(\rho_s - \rho)(1-p)} H_{s,b}^{2.5} \sin(2\alpha_b) \tag{2}$$

where ρ is the density of water, K an empirical coefficient with a recommended value of 0.39 (CERC, 1984), g is the acceleration due to the Earth's gravity, $H_{s,b}$ is the significant wave height at breaking, p is the porosity index with typical approximate value $p \approx 0.4$, α_b is the wave angle at breaking and γ_b is the breaker index.

Figure 1. (a) Location map of the study site. (b) Photo of Borth. (c) A groyne, part of the recently constructed coastal defence system.

It is worth mentioning that later studies suggested the empirical coefficient K should take an approximate value of 0.2 (Schoonees and Theron, 1993), (Schoonees and Theron, 1996). However, in this study the coefficient K was derived by the following equation (USACE, 2002):

$$K = 1.4\exp(-2.50 * D_{50}) \tag{3}$$

where D_{50} is the median grain size which for the Borth site has been taken to be 0.3mm and consequently, $K=0.66$.

By combining Equation 1 and Equation 2, and assuming angles are small, the diffusion equation can be derived:

$$\frac{\partial y}{\partial t} = G\frac{\partial^2 y}{\partial x^2} \tag{4}$$

where x is the longshore distance, y is the cross-shore position of the shoreline, and G is the shoreline diffusivity coefficient whose specific form depends upon the choice of longshore transport formula.

A solution to the diffusion equation for the case of a groyne of infinite length which blocks the littoral drift was originally provided by Pelnard-Considère (1956), after applying the appropriate initial conditions: for $t=0$, $y(x,t)=y(x,0)=0$ and the boundary conditions at x=0 (the location of the groyne): $-\frac{\partial(0,t)}{\partial x} = \tan(a_b)$, $x \in (0,+\infty)$ the following solution may be derived:

$$y(x,t) = \tan(a_b)\,(2\frac{\sqrt{Gt}}{\sqrt{\pi}}\exp\left(-\left(\frac{x}{2\sqrt{Gt}}\right)^2\right) - x erfc\left(\frac{x}{2\sqrt{Gt}}\right)) \tag{5}$$

where G is the shoreline diffusivity coefficient given in m^2/sec or ft^2/sec, and α_b the breaking wave angle in radians. However, this formula is applicable only to the ideal situation of constant breaking wave angle.

Walton and Dean (2011) presented a piece-wise analytical solution to the diffusion equation, based on a Heaviside technique, which enables the calculation of the final shoreline position due to a random sequence of waves which propagate towards the shoreline in the vicinity of an infinite groyne. The solution for three successive time steps with ($t_0 \le t \le t_3$), and for time-varying shoreline diffusivity $G(t)$ and breaking wave angle α_i is given by the following expression:

$$\begin{aligned} y_3(x,t) = {} & \alpha_1 f(G_{t0,t}, t-t_0)H(t-t_0) + \\ & -\alpha_1 f(G_{t1,t}, t-t_1)H(t-t_1) + \alpha_2 f(G_{t1,t}, t-t_1)H(t-t_1) + \\ & -\alpha_2 f(G_{t2,t}, t-t_2)H(t-t_2) + \alpha_3 f(G_{t2,t}, t-t_2)H(t-t_2) \end{aligned} \tag{6}$$

where a_1, a_2, a_3 are breaking wave angles corresponding to successive wave incidents during the corresponding time steps, H is the Heaviside function,

$$f\left(G_{ti,t}, t - t_i\right) = (2\frac{\sqrt{G_{ti,t}(t-t_i)}}{\sqrt{\pi}}\exp\left(-\left(\frac{x}{2\sqrt{G_{ti,t}(t-t_i)}}\right)^2\right) - x erfc\left(\frac{x}{2\sqrt{G_{ti,t}(t-t_i)}}\right)$$

,

$G_{ti,t}$ is the average of $G(t)$ over the interval t_{i-1} to t_i and is given by:

$$G_{ti,t} = \frac{\int_{ti}^{t} G(t)dt}{\int_{ti}^{t} dt}, \text{ simplified to: } G_{tj,tk} = \frac{\sum_{i=j}^{k} G_i \Delta t}{t_k - t_j} \tag{7}$$

in case solutions are desired at the end points of equally spaced time step integrals. The solution in Equation 6 can be extended in the obvious manner to an arbitrary number of steps. Combining Equations 5 and 6, an assessment of the effects of the blockage of the littoral drift in the vicinity of the newly constructed coastal defence system, on Borth beach was carried out.

Prior to evaluating this solution, it was necessary to transform the sequence of offshore wave conditions to the breaker point near the shore. The dataset of offshore waves comprised values of the significant wave height H_s, the wave angle α, and the peak wave period T_m, every 3 hours, for a total time span of almost 12 years). Bathymetric data was obtained from the EDINA marine digimap. A cross-shore profile corresponding to the beach of Borth was chosen. Specifically, a segment of this profile was selected with water depth at the offshore limit of 9.02 m, and water depth at the inshore limit of 1.36 m. The length of the span of this segment was 796 m and the spatial resolution was 20.4 m.

Then, a semi-empirical numerical procedure was used to calculate the wave transformation across the chosen beach profile towards the shoreline. The procedure was an interactive, stepping process, starting at the offshore limit (the position were the wave time-series were available). The waves were transformed towards the shoreline from one spatial step on the beach profile to the next, accounting for shoaling, refraction and breaking.

The procedure for every pair of consecutive spatial steps of the sellected cross-shore profile, starting from its offshore limit, is the following:

1. Calculate the refraction coefficient at the seamost point, (depth d_1), as:

$$K_1=KH_1 / d_1 \tag{8}$$

Here we have used Hunt's (1959) approximation so:

$$KH_1 \approx \sqrt{Y^2+\frac{Y}{f(Y)}};$$

$$f(Y) = (1 + 0.666666Y + 0.355555Y^2 + 0.160846Y^3 + 0.063210Y^4 + 0.021754Y^5 + 0.006541Y^6)$$

;

$$Y = \frac{v^2 \times d_1}{g};$$

$$v = \frac{2\pi}{T};$$

and T is the wave period.

2. Similarly, for the adjacent shoreward point, the refraction coefficient is calculated:

$$K_2=KH_2 / d_2$$

where d_2 the water depth at the shoreward point.

3. The wave angle at the shoreward point is calculated:

$$\varphi_2=asin(\theta_1) \ x \ 180/\pi \tag{9}$$

where:

$$\theta_1 = K_1/K_2 \times \sin(\varphi_1 \times \frac{\pi}{180})$$

φ_1: wave angle at the offshore point.

4. The wave height at the shoreward point, H_2, is calculated as:

$$H_2 = H_1 \times \sqrt{\lambda} \tag{10}$$

where λ, is the combined refraction and shoaling coefficient given by the following equation:

$$\lambda = \frac{(K_2 \times \cos(\varphi_1 \times \frac{\pi}{180}) \times (1+2\times\frac{KH_1}{\sinh(2\times KH_1)})}{(K_1 \times \cos(\varphi_2 \times \frac{\pi}{180}) \times (1+2\times\frac{KH_2}{\sinh(2\times KH_2)})} \tag{11}$$

5. Check for breaking. We used 3 different wave breaking conditions so as to conduct a sensitivity analysis. The first wave breaking condition which was to set the breaking index $\gamma_{0.78}=H_b/d=0.78$. In other words, wave heights which were exceeding 78% of the water depth *d*, were considered to break, and their height was reduced to *0.78 x d*.

   The second condition was that proposed by Battjes and Stive (1985) which specified the breaker index to be a function of offshore wave steepness:

$$\gamma_{ba}=0.5+0.4tanh(33s_d) \tag{12}$$

   where s_d is the deep-water wave steepness.

   The final condition is that suggested by Ruessink *et al.* (2003), which is not constant in the cross-shore direction but rather related to the local wave number *k* and water depth *d* (Equation 7):

$$\gamma_R=0.76kd+0.29 \tag{13}$$

This wave breaking model was developed for beaches with a single or multiple longshore bars by Ruessink *et al.* (2003) who also state it is applicable in the case of non-barred beaches (as is our case-study at Borth).

Following the transformation of the waves it is necessary to define the time interval over which waves are to be averaged in order to implement the analytical solution procedure (see Equation 7). A large interval, (say 1 month), will smooth out individual storms while a short interval, (say 1 day), rapidly makes the solution unwieldy due to the growth in the number of terms to be evaluated. In the next section we present the results of an investigation into the sensitivity of the computed beach configuration on:

1) The longshore transport formula;
2) The wave breaking formula;
3) The averaging interval.

RESULTS

A comparison of different longshore transport formulae

Three different longshore transport formulae were successively used as base for the formation of the beach model. The first one was the well known CERC formula (see Equation 2). The second one was the Kamphuis (1991), given by the following equation:

$$Q_k = 7.3H_{sb}^2 T_p^{1.5} m_b^{0.75} D^{-0.25} sin^{0.6}(2a_b) \tag{14}$$

Compared with the CERC expression, this one includes the parameters of the beach slope (m_b), the peak wave period (T_p) and the grain size D.

The third equation used is that due to Bayram et al (2007):

$$Q_B = \frac{\varepsilon}{(\rho_s-\rho)(1-p)gw_s} F\bar{V} \tag{15}$$

According to this formula, the longshore transport sediment rate is positively dependent on the incoming wave energy flux F, as well as the average longshore current speed $\bar{V}$. The rate is inversely proportional to the factor of the sediment fall velocity w_s. The transport coefficient ε expresses the proportion of wave energy which is required to keep the sediments in suspension. The remaining factors are the same as in the CERC formula.

Then, we successively coupled each one of the Equations 2, 14 and 15, which provide the longshore sediment transport rate, with the continuity of sand equation (Equation 1). Thus, 3 different forms of the analytical solution were derived, for the simulation of the shoreline evolution on the downdrift side of the representative groyne, at the beach of Borth. In order to make a comparison among the results of the simulation for each form of the analytical solution (corresponding to each one of the 3 different longshore transport formulae), the same wave breaking formula was used for the calculation of the wave transformations (breaking index $\gamma_{0.78}$=0.78) and the same temporal resolution of the forcing (1 week). Figure 2 presents the consequent results which are discussed further in the next section.

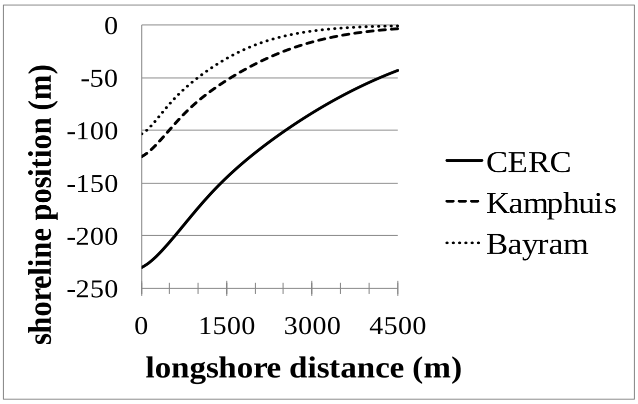

Figure 2. A comparison of the shoreline configuration due to the application of different longshore transport formulae to the beach model. In all cases, the wave breaking index was set to $\gamma_{0.78}$=0.78, and the time interval to 1week.

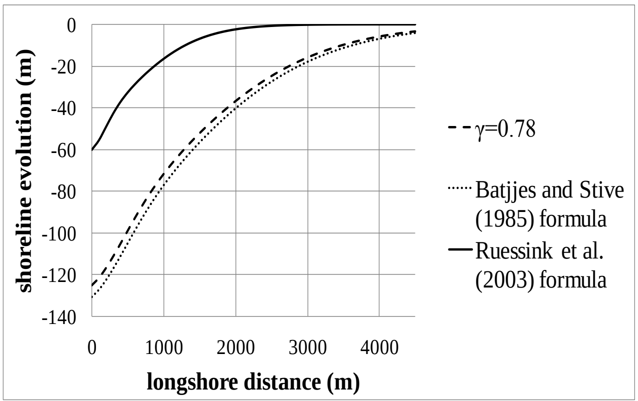

Figure 3. A comparison of the shoreline configuration due to the application of different wave breaking conditions to the beach model. In all cases, the Kamphuis (1991) longshore transport formula was applied, and the temporal resolution step was set to 1week.

The impact of the chosen wave breaking condition on the prediction of the shoreline position

As mentioned in the Methodology Section, 3 different wave breaking conditions were successively applied to the wave model, resulting in different assessments of the transformed waves to the breaker point near the shore. The fact that the sequence of the transformed waves is used as input data to the beach model, was a motivation for us to investigate the sensitivity of the beach model regarding the applied wave breaking formula. Since the CERC formula calculated a conspicuously greater rate of transported sediments on the downdrift side of the groyne (Figure 2) we chose for this application one of the other two longshore transport formulae which gave comparable results (Figure 2). This was the Kamphuis formula(Equation 14). The temporal resolution of the wave conditions was chosen to be 1 week, (ie. weekly averaged wave conditions were used in Equation 6). The resulting shoreline evolution is shown in Figure 3 for each of the three different transport formulae.

Sensitivity Analysis of the beach model regarding the temporal resolution of the forcing

Finally, a sensitivity analysis of the beach model was conducted regarding the chosen time interval; 3 different time intervals were successively applied to the beach model. The longshore transport formula which was used for this test was the Kamphuis (Equation 14). Regarding the chosen wave breaking condition, from Figure 3 it can be seen that the application of the Ruessink formula (given by Equation 13) to the beach model had as a result the loss of a relatively small volume of sediments on the downdrift side of the groyne (Figure 3). On the other hand, the application of the breaking wave index $\gamma_{0.78}$ or γ_{ba} (given by Equation 12) had approximately the same impact on the prediction of the shoreline evolution (Figure 3). Finally we chose the wave breaking condition given by Equation 12 (Battjes and Stive, 1985) for investigating the sensitivity of the beach model regarding the time interval. Results can be seen in Figure 4.

DISCUSSION

In common with several other researchers we found that the CERC formula predicts a greater longshore sediment transport rate than either of the Kamphuis (1991) and Bayram (2007) formulae (Figure 2). Interestingly, the analytical solutions based on the Kamphuis and Bayram longshore transport formulae, respectively, produced nearly the same results (Figure 2). In the future, more longshore transport rate equations could be tested against the three formulae which have been applied here. For example, the recently provided formula by Tomasicchio *et al*. (2013).

It was observed that using a simple wave breaking criterion, the wave breaking index $\gamma_{0.78}$, provided very similar results to those obtained using the Battjes and Stive (1985) wave breaking coefficient γ_{ba}. This is notable, since in contrast with the coefficient $\gamma_{0.78}$ which is not only spatially constant but also the same for each incident wave condition, the Battjes and Stive formula (Equation 12), is varying in accordance with the wave steepness of the corresponding incident offshore wave. Figure 3 shows that the application of these two wave breaking conditions had the same impact on the beach models' results. Finally, the breaking wave model by Ruessink et al (2003) exhibited smaller values of the coefficient γ (denoted in this case by

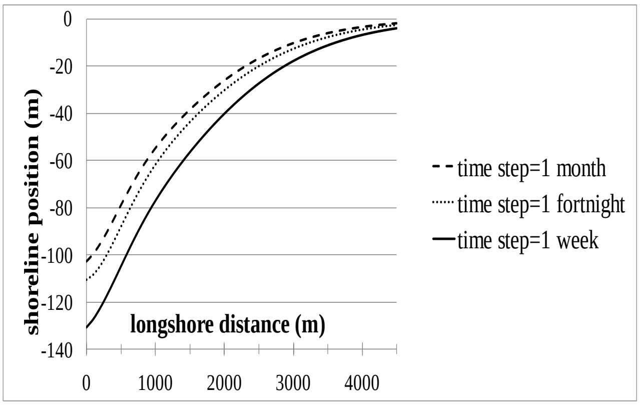

Figure 4. A comparison of the shoreline configuration due to the application of different temporal resolution of the forcing to the beach model. In all cases, the wave breaking index γ_{ba} was applied, and the Kamphuis longshore transport formula.

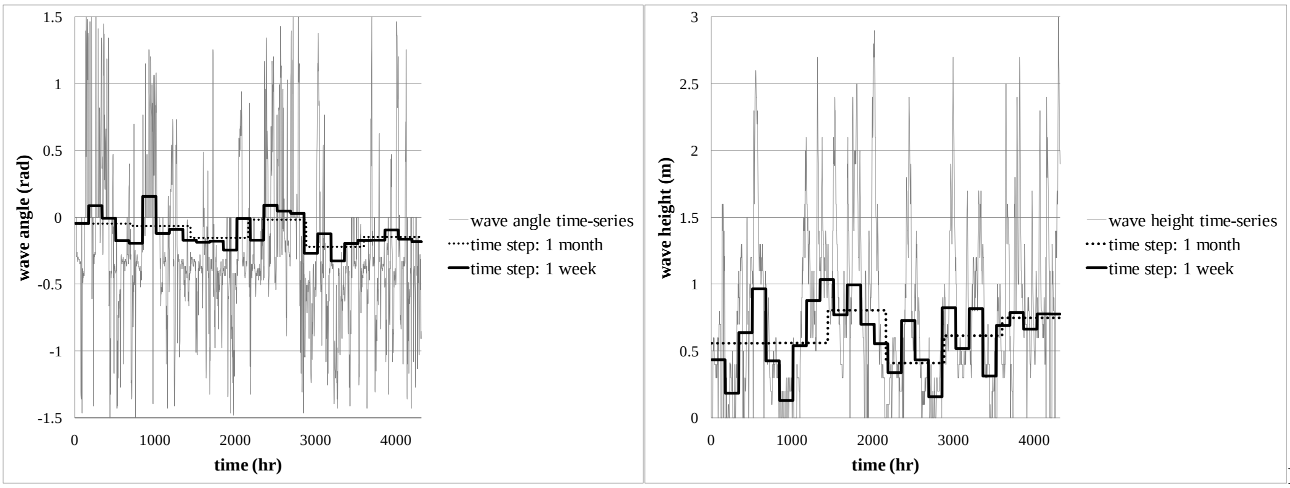

Figure 5. (a) Averaged wave angle for time step: 1 month; 1 week and comparison with the wave angle time-series. (b) Averaged wave height for time step: 1 month; 1 week and comparison with the wave height time-series.

γ_R, and given by Equation 13). The application of the breaking wave coefficient γ_R on the beach model caused significantly less erosion on the downdrift side of the groyne, as can be seen in Figure 3.

The sensitivity to the time interval proved to be very important (Figure 4). This can be explained by the fact that as the temporal resolution of the forcing increases (from 1 month to 1 week), the averaged wave angle for each direction of the corresponding littoral drift, increases as well (Figure 5a). Since the dominating direction of the propagating waves is the southwest (considered negative in Figure 5a), it is expected greater erosion to be observed on the downdrift side of the groyne, as the value of the time step is decreasing. Similarly, more intervals corresponding to storms (high wave height) were taken into account, with the decrease in the value of the temporal step (Figure 5b).

In addition, convergence of the solution, as the number of temporal steps increases, proved to be not feasible, especially near the groyne. This is illustrated in Figure 6, where the beach profile has been plotted for different temporal resolutions of the forcing. The value of the temporal step has been gradually shifted from 1 month to 3 hours (the latter is equal to the time step of the available hindcast data). Since the computational effort increases with the number of steps, ensemble results were determined by running the model in six batches of six-month spans where hindcast data were available, (Figure 6).

It is worth noting that the structure of the Heaviside solution means it becomes increasingly unwieldy with greater numbers of time steps. The sensitivity of the solution to the temporal resolution of the wave conditions is a serious constraint. The approach of Zacharioudaki and Reeve (2008) is more complicated in form but does not suffer from this deficiency.

CONCLUSIONS

An analytical model based on a Heaviside technique was used in this study for the preliminary assessment of the impact of a random sequence of waves on a shoreline located in the vicinity of an infinite and impermeable groyne. As a case study was used the beach of Borth in Wales, UK, where a recently constructed coastal defence system was represented by a single infinite groyne, and the consequent blockage of the littoral drift was assessed, using a hindcast wave time-series (including 3 hourly data regarding the wave height H_s, wave angle α, and period T_m), covering a period of approximately 12 years. An inshore wave time-series has been derived by transforming the hindcast deep water waves accounting for refraction, shoaling and breaking, before being used as input data for the beach model.

Three different longshore sediment transport rate formulae were independently applied to the beach model. The well-known CERC longshore sediment transport rate formula predicted a significantly greater transport rate than the other two, and consequently, more accretion on the updrift side of the groyne. On the other hand, the Kamphuis and Bayram formulae resulted in comparable predictions of the final shoreline position.

In addition, three different wave breaking conditions were successively applied to the wave model, altering the significant heights of the transformed waves which were used as input data to the beach model. The breaking wave model expressed by the constant cross-shore coefficient: $\gamma_{0.78}$, proved to give very similar results to those found with the Battjes and Stive (1985) wave breaking formula, although this equation proposes a weak dependence of the wave breaking index γ_{ba} on the time varying wave steepness of the offshore waves. In contrast, the Ruessink *et al.* (2003) breaking wave formula, which is spatially varying in

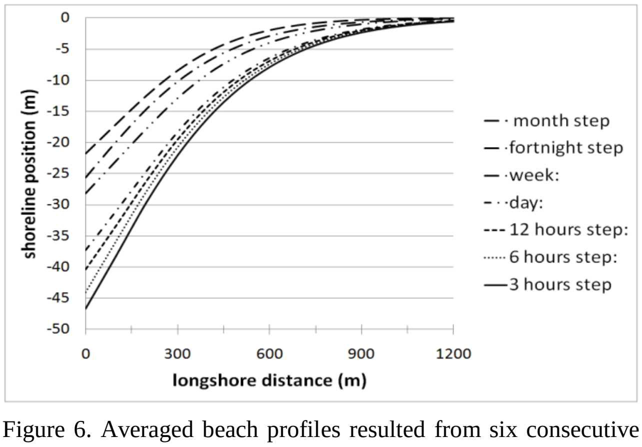

Figure 6. Averaged beach profiles resulted from six consecutive applications of the beach model. In all cases, the same simulation period of 6 months was chosen; the wave breaking index γ_{ba} was applied as well as the Kamphuis longshore transport formula.

accordance with the local wave number k, and the depth d, reduced the broken wave heights significantly, resulting in reduced sediment transport and a conspicuously smaller amount of accretion on the updrift side of the groyne (Figures 3).

Surprisingly, the beach model proved to be especially sensitive to the chosen temporal resolution of the wave record. Obviously, the higher the temporal resolution is chosen, the more accurate the results of the beach model become. However, there is a practical limitation to the maximum temporal resolution we can achieve, due to the increasingly unwieldy nature of the Heaviside solution as the number of time steps increases.

We conclude that the beach evolution model based on the Heaviside solution method provides a useful tool for investigating short-term beach evolution. The approach also allows alternate sediment transport formulae to be incorporated and results compared. Our computed solutions demonstrated sensitivity to wave transformation and in particular the representation of wave breaking. Further, the results showed a strong sensitivity to the temporal resolution of the wave conditions, which needs to be appreciated when interpreting the simulation of beach evolution with this approach.

ACKNOWLEDGEMENT

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